# Scaling Behavior of Transverse Kinetic Energy Distributions in Au+Au Collisions at $\sqrt{s_{\mathrm{NN}}} = 200$ GeV

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With the experimental data from STAR on the centrality dependence of transverse momentum $p_T$ spectra of pions and protons in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, we investigate the scaling properties of transverse energy $E_T$ distributions at different centralities. In the framework of cluster formation and decay mechanism for particle production, the universal transverse energy distributions for pion and proton can be described separately but not simultaneously.

## I. INTRODUCTION

Recently, the scaling behaviors of the transverse momentum distributions for particles produced in high energy collisions attract more and more attentions. The search on the scaling behaviors of particle spectrum is significantly important for understanding the evolution of high energy heavy ion collisions and particle production mechanisms, because the produced particle distribution carries information about the dynamics of the system and is one of the most important observables in high energy collisions. In [1, 2, 3], with the experimental data from STAR, PHENIX and BRAHMS, we found there is a scaling behavior of transverse momentum $p_T$ spectrum for pions. The scaling function is independent of the colliding system, the colliding energy, the centrality and (pseudo)rapidity. For protons and anti-protons, there also exists a similar scaling behavior which is independent of the centrality and rapidity in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV [4]. For strange particles, such as $K$, $\Lambda$ and $\Xi$, the scaling behaviors for their distributions are under consideration.

In high energy heavy ion collisions, a lot of the initial kinetic energy of the bombarding nuclei is converted into those of the produced particles, including longitudinal and transverse ones. Different from momentum, kinetic energy is a scalar and is directly associated with the temperature of the hot medium created in the collisions. For different species of particles, the same momentum corresponds to different kinetic energy because of mass effect. As well-known, the kinetic energy, rather than momentum, of particles in a thermalized system satisfies the Boltzmann distribution. Thus the distribution of kinetic energy of particles produced in ultra-relativistic heavy ion collisions is more effective in revealing the thermal properties of the system. With above consideration in mind, one can ask whether the transverse kinetic energy $E_T$ distribution has similar scaling behavior. This is our motivation of investigating the scaling properties of distributions of transverse kinetic energy $E_T$ of particles. We hope the mass effect can be suppressed in the new scaling function.

In this paper the scaling $E_T$ distribution of protons in the mid-rapidity region is studied and compared with the scaling $p_T$ distribution. It is very essential to ask why the scaling behaviors exist for different particles and what is the potential universal dynamics. In [4] we found that the string overlap mechanism can not explain data for pion and proton simultaneously. In this paper we will consider another mechanism based on the parton percolation theory [5, 6, 7], and hope to get more information about particle production mechanism in nuclear collisions.

This paper is organized as follows: In Sec. II, we discuss relation between distributions for $E_T$ and $p_T$, and the scaling property of $E_T$ distribution of protons produced in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Universal scaling functions for pion and proton are given. In Sec. III we discuss the universal scaling functions in the framework of cluster decay. The conclusion is drawn in Sec. IV.

## II. THE SCALING BEHAVIOR OF TRANSVERSE ENERGY DISTRIBUTION

From the definition of transverse kinetic energy $E_T \equiv m_T - m_0 = \sqrt{p_T^2 + m_0^2} - m_0$, with $m_0$ mass of the particle, the invariant $E_T$ distribution is

$$\frac{d^2N}{E_T dE_T dy} = (1 + \frac{m_0}{E_T})\frac{d^2N}{p_T dp_T dy} \ . \qquad (1)$$

From the definition of $E_T$, one can see that when $m_0 \to 0$ or $p_T \to \infty$, $E_T$ is approximately equal to $p_T$, and

$$\frac{d^2N}{E_T dE_T dy} \approx \frac{d^2N}{p_T dp_T dy} \ . \qquad (2)$$

So it is feasible to only consider the scaling behavior of $p_T$ distribution in the range $p_T > 0.5$ GeV/$c$ for pions, whose mass is only 0.139 GeV. Actually, the results obtained in [1, 2, 3] are excellent. But if $m_0$ is large, as for kaons, protons and anti-protons, etc, we must consider the mass effect and investigate the scaling behaviors of $E_T$ distributions in the same range of $p_T$.

In [4] we have summarized the method for searching the scaling behavior of the particle's $p_T$ spectrum. Now

we can use the similar method to investigate the scaling behavior of the transverse energy distribution of final state particles, with transverse kinetic energy $E_T$ instead of transverse momentum $p_T$. First, define a scaled variable

$$z = E_T/K , \qquad (3)$$

and the scaled spectrum

$$\Phi(z) = A \left. \frac{d^2N}{2\pi E_T dE_T d\eta} \right|_{E_T=Kz} , \qquad (4)$$

with $A$ and $K$ free parameters chosen to fit the scaled distributions to the same curve. Of course values of $A$ and $K$ are different for distributions at different centralities, given that for most central collisions they are set to be 1. Then $\Phi(z)$ can be regarded as a parameterization of the $E_T$ distribution in most central Au+Au collisions. To obtain a universal scaling function for all centralities, independent of the arbitrary in choosing values of $A$ and $K$ for the most central collisions, we introduce another scaled variable

$$u = z/\langle z \rangle = E_T/\langle E_T \rangle , \qquad (5)$$

and the normalized scaling function

$$\Psi(u) = \langle z \rangle^2 \Phi(\langle z \rangle u)/\int_0^\infty \Phi(z)zdz . \qquad (6)$$

Here $\langle z \rangle$ is defined as

$$\langle z \rangle \equiv \int_0^\infty z\Phi(z)zdz/\int_0^\infty \Phi(z)zdz , \qquad (7)$$

and one can easily check that $\int duu\Psi(u) = 1$ and $\langle u \rangle = \int duu^2\Psi(u) = 1$. With above steps, one can investigate whether there exists a scaling behavior of transverse energy distribution for protons produced at mid-rapidity in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, which has a wide rang $p_T$ coverage [8]. As shown in Fig. 1, all data points for different centralities can be shifted to the same curve within errorbar, and the corresponding values of parameters $A$ and $K$ shown in TABLE I. To parameterize the curve, we define $v = \ln(1 + z)$, and the scaling function is

$$\Phi(z) = 18.604 \exp(-0.6765v - 6.48v^2 + 1.477v^3) . \quad (8)$$

After normalization,

$$\Psi(u) = 1.4846 \exp(2.394v - 5.79v^2 + 0.977v^3) , \quad (9)$$

with redefined $v = \ln(1 + u)$.

In order to see the agreement between the scaling distribution and the data in linear scale, a ratio $B$ can be defined as
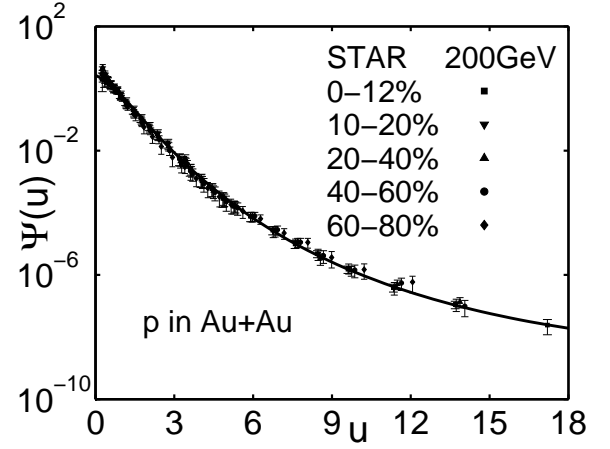
B=experimental data/fitted results.



FIG. 1: Scaling behavior of the $E_T$ spectra for protons produced at mid-rapidity in Au+Au collisions at RHIC. Feed-down corrections are considered in the data. The solid curve is from Eq. (9). The data are taken from [8] after calculation with Eq. (1).

| STAR | $\pi$ | | $p$ | |
|---|---|---|---|---|
| centrality | K | A | K | A |
| 0-12% | 0.4192 | 0.0038 | 0.5786 | 0.1440 |
| 10-20% | 0.4293 | 0.0059 | 0.5769 | 0.1788 |
| 20-40% | 0.4428 | 0.0114 | 0.5710 | 0.3022 |
| 40-60% | 0.4535 | 0.0307 | 0.5635 | 0.6732 |
| 60-80% | 0.4571 | 0.1025 | 0.5437 | 2.0985 |
| 40-80% | 0.4536 | 0.0466 | | |

TABLE I: Parameters for coalescing all data points to the same curves in Figs. 1 for $p$ and 4 for $\pi^+$.

First let us look at the agreement of the fit to the $p_T$ distribution. The fitted normalized $p_T$ scaling function of proton is obtained from [4], which is given as follows,

$$\Psi(u) = 0.064 \exp(13.6v - 16.67v^2 + 3.6v^3) , \qquad (10)$$

with $v = \ln(1 + u)$ and $u = p_T/\langle p_T \rangle$. The result of $B$ for the deviation of the scaling $p_T$ function from data in most central collisions is shown in Fig. 2. They agree with each other within experimental error. Then one can investigate $B$ for the $E_T$ case. The result is shown in Fig. 3. As can be seen in Figs. 2 and 3, in the middle region, both fits are good. But at the low and high region, the agreement of the scaling $E_T$ distribution is better. This indicates that the $E_T$ scaling can describe the experimental data better. For data at other centralities, similar conclusion can be made.
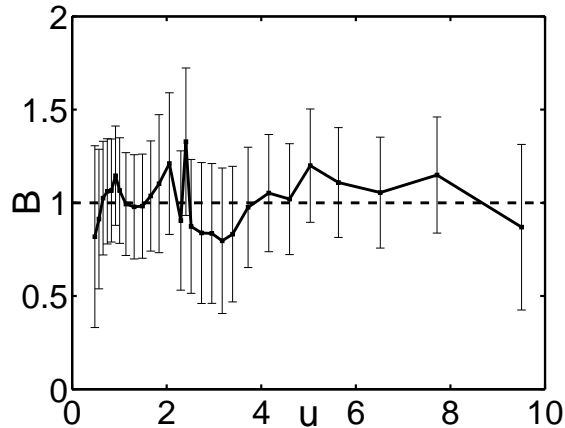
FIG. 2: The ratio $B$ of the experimental data to the scaling $p_T$ fitted results Eq. (10), here $u = p_T/\langle p_T \rangle$. Errorbars shown are calculated from those from experiment with the definition of $B$.
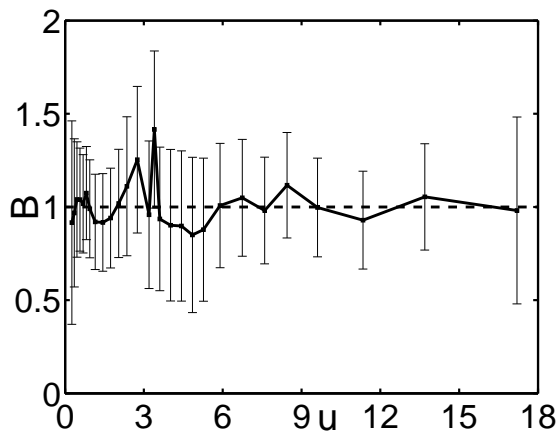


FIG. 3: The ratio $B$ of the experimental data to the scaling $E_T$ fitted results Eq. (9), here $u = E_T/\langle E_T \rangle$.

## III. SCALING TRANSVERSE KINETIC ENERGY DISTRIBUTION AND CLUSTER DECAY

In [1, 2, 3, 4] and last section, we have found the scaling laws of transverse momentum and kinetic energy spectra for pions, protons and anti-protons . With those results, we should try to trace out the particle production mechanism. In [4] we have shown that string picture for particle production can not describe the scaling properties of $p_T$ distributions for both pions and protons simultaneously, because from the pion and proton spectra one can get opposite changes of string overlap degree from central to peripheral collisions.

In the framework of parton percolation [5, 6, 7], massive color-neutral clusters are formed in high energy collisions from produced partons. In this picture all observed final state particles are decay products of those clusters with different sizes. Now we discuss whether such a picture can describe the transverse energy distribution. One can use the inverse of transverse kinetic energy squared, $x$, to describe the cluster size with distribution denoted by $W(x)$. Here the transverse kinetic energy is used in place of transverse momentum in [5, 6, 7]. The normalized $W(x)$ has been supposed to be a Gamma distribution

$$W(x) = \frac{\gamma}{\Gamma(k)}(\gamma x)^{k-1}\exp(-\gamma x) , \qquad (11)$$

with $\gamma$ and $k$ two parameters. To get the spectra of final state particles one needs to know the cluster fragmentation function $f(x, E_T)$ for each species of particles. Here $f(x, E_T)$ gives the probability of producing a hadron with transverse kinetic energy $E_T$ from a cluster of size $x$. We have no first-principle as an instruction for the functional form of fragmentation function. We expect that the fragmentation function is a function of the fraction $z = E_T\sqrt{x}$ of transverse kinetic energy of the produced particle relative to that of a cluster with size $x$. This fraction is a close analogue of that for usual fragmentation functions from hard partons to hadrons. For this reason, one can assume that the fragmentation function for cluster decay takes the same functional form as the usual ones, as used in [9] and references therein,

$$f(z) = Dz^a(1 - z)^b(1 + cz^d) \qquad (12)$$

with $D$, $a$, $b$, $c$ and $d$ five parameters.

With the cluster size distribution and fragmentation function, the $E_T$ distribution of a species of final state particles can be expressed as

$$\frac{dN}{E_T dE_T} = C \int_0^{1/E_T^2} dx f(E_T\sqrt{x}) \times W(x) . \qquad (13)$$

In last equation $C$ is the normalization constant for the total number of clusters formed before hadronization. In real fitting parameters $C$ and $D$ always appear as a product. So one can absorb $D$ into $C$. The upper limit in the integration is given from the consideration that hadron's fraction of transverse kinetic energy cannot be larger than 1. The physics behind above formula is as follows. The clusters are formed before their decay, so that $W(x)$ is the same for all species of final state hadrons. On the other hand, $f(z)$ is different for different hadrons, because different hadrons come from different fragmentation channels of clusters, but $f(z)$ should have no connection with the cluster size $x$, thus is independent of the colliding system, centrality and rapidity, namely universal for all collision processes. This is also a parallel to the usual parton fragmentation functions which are independent of the processes for the hard parton production. It is easy to show that last equation guarantees scaling of the particle distribution. Under the transformation $x \to \lambda x$, $\gamma \to \gamma/\lambda$ and $E_T \to E_T/\sqrt{\lambda}$, both the two functions

$W(x)$ and $f(z)$ are all invariant, so the $E_T$ distribution is also invariant within a normalization constant. This invariance means that a change of the mean transverse kinetic energy can be equilibrated by a change of the value of parameter $\lambda$ for the cluster distribution. Thus it shares the essence with the well-known renormalization group transformation. This invariance is the scaling we are looking for.
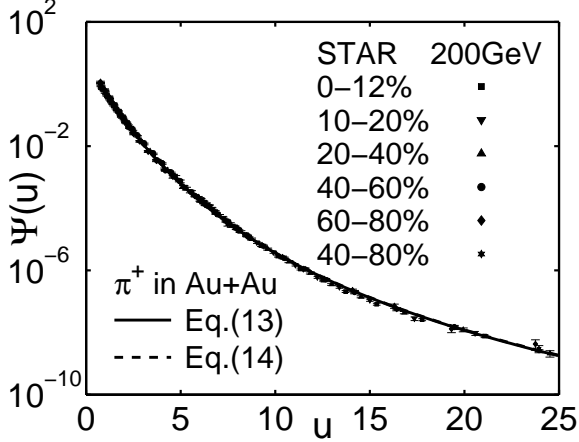


FIG. 4: Normalized scaling distribution for pions produced at mid-rapidity in Au+Au collisions at RHIC with scaling variable $u = p_T/\langle p_T \rangle$. Feed-down corrections also are considered in the data. The solid curve is from Eq .(13), and the dash curve is from Eq .(14). The data is from [8].

Now we show how to get the $E_T$ distributions of pions and protons from the decay of clusters. Because pion is very light, $E_T \simeq p_T$ in almost the whole range of observed transverse momentum, one only need consider the scaling $p_T$ distribution. We use a recent set of data in [8] for pion. Considering $p_T$ range in the data used now is much wider than that used in [1], we fit the new data. All data points can be put to the same curve by suitable $A$ and $K$, whose values are tabulated in TABLE I. The new parameterization for the normalized scaling function is now

$$\Psi(u) = 0.288 \left(1 + \frac{u^2}{7.868}\right)^{-4.3} (1 + 13.6e^{-1.9u}), \quad (14)$$

with $u = p_T/\langle p_T \rangle$. As shown in Fig .4, the agreement between the scaling function and data is great. To describe the scaling function for pion with the cluster mechanism, one can work with Eq. (13) and fit the obtained scaling function with 7 parameters, $C, \gamma$ and $k$ for cluster distributions, $a, b, c$ and $d$ for the fragmentation function from a cluster to pion. The fitted parameters are tabulated in TABLE II. For comparison, the fitted curve from the cluster decay is also plotted in Fig. 4. Two curves for the scaling function and the fitted result cannot be distinguished in the plot in log scale. For protons, one should consider the scaling $E_T$ distribution, as discussed in last

section. To fit proton's scaling function, one cannot treat all the three parameters for the cluster distribution as free parameters. As will be addressed below, parameter $k$ must be the same in fitting the scaling functions for pion and proton. The fitted parameters for proton case are also given in TABLE II, and the fitted curve is shown in Fig. 5. The normalized scaling $E_T$ function Eq .(9) is shown in this figure too. As can be seen from Fig. 5, the fit is also beautiful in a wide range of $E_T$. The inset in Fig. 5 gives the ratio of the scaling function for proton to the fitted distribution from cluster decay. It should be noted that the mean transverse momenta are different for pion and proton in the collisions with given centrality. In obtaining the scaling functions we try to scale the momentum (or transverse kinetic energy) to a variable with unit mean. For this purpose, the scaling factors for pions and protons must be different. The scale in $p_T$ or $E_T$ can be translated into that of $x$ and finally to $\gamma$, as discussed in the paragraph following Eq. (13). So the values of $\gamma$ for pions and protons obtained in our fitting are different. This difference in scaling factor also causes different values of $C$, as shown in TABLE II. Another reason for the difference of values of $C$ for pion and proton is that $C$ contains in fact parameter $D$ as a factor which should be much smaller for proton than for pion because of the low yield of proton relative to pion. Besides, the distribution of clusters $W(x)$ is the same for all species of final state particles, so one can figure out $k$ is the same for pions and protons, because the scale change in $p_T$ or $E_T$ does not affect $k$. Because the parameter $a$ for pion is about $-1$ while that for proton is only about 0.2, the density of pion with low transverse kinetic energy (or momentum) is much higher than for proton, in agreement with experimental fact that $p/\pi \sim 0$ as $p_T \to 0$. Now we can say the mechanism of cluster formation and decay can describe the universal scaling behaviors of transverse energy distributions for either pion or proton.

But can the cluster mechanism describe all those final state particle spectra at the same time, as a true particle production mechanism should be able to? The answer lies in the trends of change of the parameter $K$ for pion and proton from central to peripheral collisions. $K$ increases for pions but decreases for protons. The same results are also obtained in earlier works [1, 2, 3, 4] for the $p_T$ distributions. Such trends are closely associated with the fact that the pion spectrum is suppressed from intermediate to high $p_T$ in central Au+Au collisions while no suppression was observed for the proton spectrum. In fact, from Eq. (5) one can see that the values of $K$ used in shifting data points to the normalized scaling distribution are the mean values of transverse kinetic energy $\langle E_T \rangle$. With the cluster mechanism, on the other hand, $\langle E_T \rangle$ can be calculated from Eq. (13) as

$$K = \langle E_T \rangle = \frac{\int_0^\infty dx W(x)/x^{3/2}}{\int_0^\infty dx W(x)/x} \frac{\int_0^1 dz z^2 f(z)}{\int_0^1 dz z f(z)} . \quad (15)$$
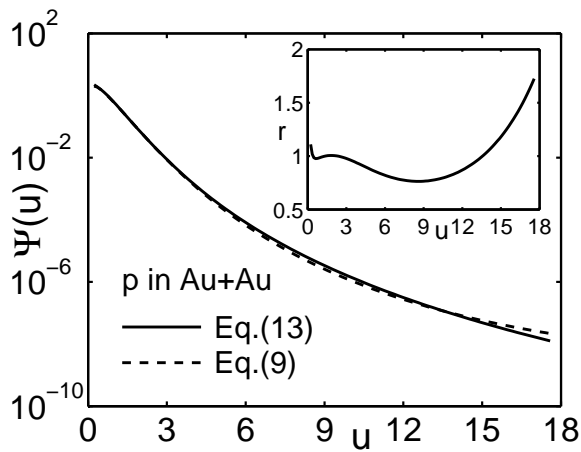
FIG. 5: Comparison between the normalized scaling distribution for protons produced at mid-rapidity in Au+Au collisions at RHIC and the fitted result from the cluster decay mechanism.

tons in Au+Au collisions at 200 GeV. We showed that the $E_T$ scaling behavior can describe experimental data

|   | $\pi$ | $p$ |
|---|---|---|
| $C$ | 12.7766 | 0.3857 |
| $\gamma$ | 203.7292 | 318.4096 |
| $k$ | 4.3164 | 4.3164 |
| $a$ | -1.0842 | 0.1890 |
| $b$ | 10.0671 | 25.9110 |
| $c$ | 201.0009 | 142.9827 |
| $d$ | 0.0016 | 0.5406 |

TABLE II: Values of parameters of the universal transverse energy distributions Eq. (13) for pions and protons.

As stated above, cluster distribution $W(x)$ is the same for different species of particles, while cluster fragmentation functions $f(z)$ are the same for a species of particle produced, independent of colliding centrality. If we take a ratio between values of $K$ for different centralities, the ratio must be the same for all particles, since the $f(z)$ terms cancelled in the ratio. This demands that values of $K$ for different species of particles must change in the same way with centrality. This demand cannot be satisfied in the cluster mechanism. This fact indicates that the cluster formation and decay mechanism is not a universal one for the production of final state pions, protons, and other particles in high energy collisions. As shown in [7], however, one can describe spectra of both pion and (anti)proton with different values of the parameter $k$ in the cluster distribution. Different values of $k$ implies that pions and (anti)protons originate from clusters with different distributions.

Finally a brief discussion on the distributions at LHC energies can be addressed. In Pb+Pb collisions at LHC energies $\langle E_T \rangle$ should be much larger than that at RHIC energies. From the universal distribution discussed in this paper, the value of $\lambda$ in the cluster distribution must be much smaller, which means that the clusters formed in those collisions at LHC energies must have mean transverse kinetic energy much larger than that at RHIC energies.

## IV. CONCLUSION

In this paper we discussed the scaling properties of the transverse kinetic energy distributions for pions and pro-

better than that for the $p_T$ case. Although the cluster formation and decay mechanism can describe spectra of either pion or proton at different centralities, it cannot give consistent description, with the same parameters in the cluster distribution, for both species of particles at the same time. Thus it can be excluded as a consistent particle production mechanism. Other particle production mechanism must be in effect for the scaling behavior.

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